NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS (EXTENSION 1)

2014 HSC Course Assessment Task 4

Friday August 15, 2014

General instructions

- Working time – 55 minutes. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided (on page 2)

SECTION II

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: .... **# BOOKLETS USED:** .....

Class (please ✔)

- ◯ 12M3A – Mr Zuber
- ◯ 12M3B – Mr Berry
- ◯ 12M3C – Mr Lowe
- ◯ 12M4A – Ms Ziaziaris
- ◯ 12M4B – Mr Lam
- ◯ 12M4C – Mr Ireland

Marker's use only.

QUESTION	2-1	5	6	7	Total	%
MARKS	$\overline{4}$	$\overline{12}$	$\overline{10}$	$\overline{11}$	$\overline{37}$	

Section I

4 marks
Attempt Question 2 to 1
Mark your answers on the answer grid provided.

Questions **Marks**

1. What is the sum of the coefficients in the expansion of $(1+x)^n$? **1**

(A) 0 (C) 2^n

(B) 1 (D) None of the above

2. Which of the following is an equation for simple harmonic motion? **1**

(A) $x = a\sin(nt+\alpha)$ (C) $x = a\cos nt + b\sin nt$

(B) $x = a\cos nt$ (D) All of the above

3. What is the coefficient of x^3 in the expansion of $(1-x)^6$? **1**

(A) 20 (C) -20

(B) 15 (D) None of the above

4. Which of the following are defining conditions for projectile motion? **1**

(A) $\ddot{x} = -g$, $\ddot{y} = 0$ (C) $\ddot{x} = 0$, $\ddot{y} = 0$

(B) $\ddot{x} = 0$, $\ddot{y} = -g$ (D) $\ddot{x} = -g$, $\ddot{y} = -g$

Answer grid for Questions 2 - 1

1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ

Examination continues overleaf...

Section II

33 marks
Attempt Questions 5 to 7
Write your answers in the writing booklets supplied. Additional writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (12 Marks) Commence a NEW page. **Marks**

(a) An object is moving with displacement x, velocity v and time t. Show that **3**

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$$

(All working must be shown to obtain full marks)

(b) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -2e^{-x}$, where x is the displacement from the origin. Initially, the particle is at the origin with a velocity of $2\,\text{ms}^{-1}$.

i. Show that $v = 2e^{-\frac{x}{2}}$. **3**

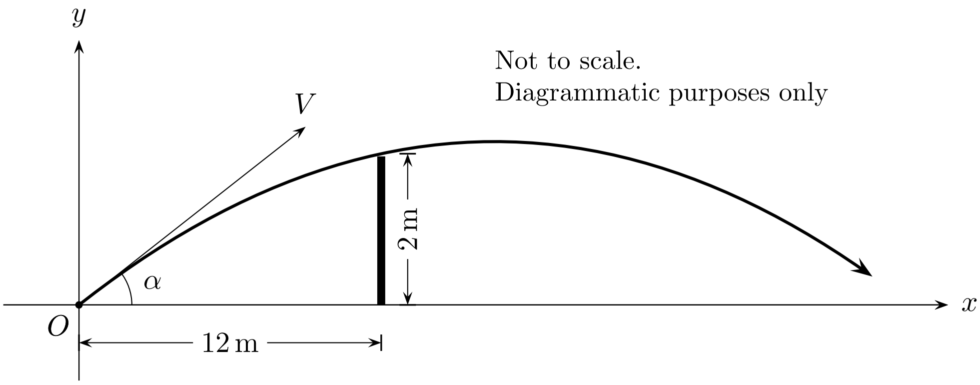
ii. Describe the behaviour of the particle's velocity as x continues to increase. **1**

(c) A particle is moving in simple harmonic motion. At the end points of the motion, the acceleration is $\pm 1\,\text{ms}^{-2}$. When the particle is $3\,\text{cm}$ from the centre of motion, the speed is $2\sqrt{2}\text{cm}\,\text{s}^{-1}$. **5**

Find the period and amplitude of the motion.

Question 6 (10 Marks) Commence a NEW page. **Marks**

A ball is thrown so that it just clears a wall 2 m high, 12 m from the thrower after 1 s. Take $g = 10\,\mathrm{ms}^{-2}$.



(a) Derive the equations of motion for this projectile. **2**

(b) Find the initial projection speed V of the ball, to the nearest metres per second. **2**

(c) Find the angle α at which the ball was thrown, correct to the nearest degree. **2**

(d) Find the maximum distance that the ball could be thrown with the same projection speed, such that it would also clear the wall. **3**

You may use the rounded off value from part (b).

(e) Copy or trace the diagram above into your working booklet. **1**

Show also on the diagram, the altered trajectory if the ball passes through a thin sheet placed above the wall, which reduces its velocity.

Question 7 (11 Marks) Commence a NEW page. **Marks**

(a) Find the term independent of x in the expansion of **3**

$$\left(\frac{x^2}{2} - \frac{3}{x^3}\right)^{10}$$

(b) Find the greatest coefficient in the expansion of $(3 + 2x)^{12}$. **4**

(c) In the expansion of $(1 + x)(a - bx)^{12}$, the coefficient of x^8 is zero. **4**

Find the value of $\dfrac{a}{b}$ in simplest form.

End of paper.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \qquad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \qquad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \qquad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \qquad a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE: $\ln x = \log_e x, \; x > 0$

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Suggested sol^{ns}

(1) D (2) A (3) B (4) C 1 each

(5) a) LHS $= \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

$$= \frac{d}{dv}\left(\frac{1}{2}v^2\right) \times \frac{dv}{dx} \quad \text{— (1)}$$

$$\left.\begin{aligned} &= v\,\frac{dv}{dx} \\ &= \frac{dx}{dt} \times \frac{dv}{dx} \end{aligned}\right\} \quad \text{— (1)}$$

$$= \frac{dv}{dt} = \ddot{x} = \text{RHS} \quad \text{— (1)}$$

b) i) $\ddot{x} = -2e^{-x}$

ie $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = -2e^{-x}$ — (1)

$$\frac{1}{2}v^2 = 2e^{-x} + c$$

$v = 2 \quad x = 0$ — (1)

$$2 = 2 + c$$

$$c = 0$$

$$\therefore \frac{1}{2}v^2 = 2e^{-x}$$

$$v^2 = 4e^{-x}$$

$$v = \pm 2e^{-\frac{x}{2}} \quad \text{(initially } v > 0\text{)} \quad \text{— (1)}$$

$$v = 2e^{-\frac{x}{2}}$$

ii) As $x \to \infty$ $v \to 0$ — (1)

(c) $\ddot{x} = -n^2 x$

$|-n^2 a| = 1$

$n^2 a = 1 \qquad a > 0$

$a = \frac{1}{n^2}$

$v^2 = n^2(a^2 - x^2)$

$8 = n^2(a^2 - 9)$ —— ①

$8 = \frac{1}{a}(a^2 - 9)$

$8a = a^2 - 9$ —— ①

$a^2 - 8a - 9 = 0$

$(a-9)(a+1) = 0$

$a = -1$ or $a = 9$ but $a > 0$ —— ①

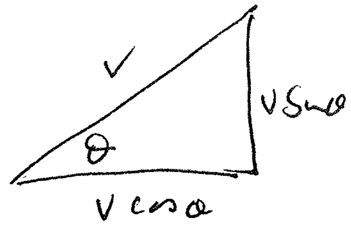
$\therefore a = 9$

so $n^2 = \frac{1}{9}$

$n = \frac{1}{3}$ —— ①

$P = \frac{2\pi}{n} = \frac{2\pi}{\frac{1}{3}} = 6\pi$ —— ①

6 a) $\ddot{x} = 0$ $\qquad$ $\ddot{y} = -10$



$\dot{x} = c_1$ $\qquad$ $\dot{y} = -10t + c_2$

$\boxed{t=0 \quad \dot{x} = V\cos\theta \quad \dot{y} = V\sin\theta}$

$\therefore c_1 = V\cos\theta$ $\qquad$ $c_2 = V\sin\theta$

$\dot{x} = V\cos\theta$ $\qquad$ $\dot{y} = -10t + V\sin\theta$

$x = Vt\cos\theta + c_3$ $\qquad$ $y = -5t^2 + Vt\sin\theta + c_4$

$\boxed{t=0 \quad x=0 \quad y=0}$ $\qquad$ $c_4 = 0$

$c_3 = 0$ $\qquad$ $y = -5t^2 + Vt\sin\theta.$

$x = Vt\cos\theta$

2

b) $t=1$ $\quad 12 = V\cos\theta$ $\quad 7 = V\sin\theta$

$\tan\theta = \frac{7}{12}$

$\theta = 30°15'$

$\therefore V = \frac{12}{\cos 30°15'} = 14\,m/s$ — 2

c) $\theta = 30°15' = 30°$ (nearest degree) — 2

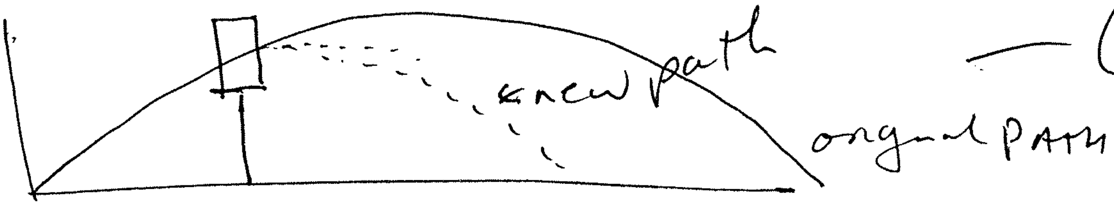
d) $y = 0$ $\quad x = \frac{V^2 \sin\theta\cos\theta}{5}$ — 1

$\theta = 45$ $\quad V = 14$ — 1

$x = \frac{14^2 \sin 45 \cos 45}{5}$

$= 19.6\,m.$ — 1

e)



— 1

(7) a $\left(\frac{x^2}{2} - \frac{3}{x^3}\right)^{10}$ $T_{r+1} = {}^{10}C_r\left(\frac{x^2}{2}\right)^{10-r}\left(\frac{-3}{x^3}\right)^r$

independent term when $x^{20-2r} \times x^{-3r} = x^0$ — 1

$20 - 5r = 0$ — 1

$r = 4.$

i.e. $T_5 = {}^{10}C_4\left(\frac{x^2}{2}\right)^6\left(\frac{-3}{x^3}\right)^4$

~~coeff~~ $= {}^{10}C_4\left(\frac{1}{2}\right)^6(-3)^4$ $\left[\frac{8505}{32}\right]$ — 1

b) $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r} \times \frac{b}{a}$ — 1

$= \frac{13-r}{r} \times \frac{2}{3}$

$= \frac{26-2r}{3r}$ — 1

$\frac{T_{r+1}}{T_r} \geq 1$ when $\frac{26-2r}{3r} \geq 1$ — 1

$26-2r \geq 3r$

$5r \leq 26$

$r \leq 5.2$

$r = 5.$

$\therefore$ Greatest coefficient $T_6 = {}^{12}C_5(3)^7(2)^5$ — 1

c) $(1+x)(a-bx)^{12}$

Coefficient of x^8 is

$${}^{12}C_8 (-b)^8 (a^4) + {}^{12}C_7 (-b)^7 (a^5) = 0 \qquad -(2)$$

$${}^{12}C_8\, a^4 b^8 = {}^{12}C_7\, a^5 b^7$$

$$\frac{a}{b} = \frac{{}^{12}C_8}{{}^{12}C_7} = \frac{\frac{12!}{4!\,8!}}{\frac{12!}{5!\,7!}} \qquad -(1)$$

$$= \frac{5}{8} \qquad -(1)$$